## 1 Introduction

- Termination criteria decide on the end of an evolutionary search process (EA).
- Performance of an EA also depends on the determination of the appropriate point in time to terminate the search.

Problem:
What is the appropriate point in time to terminate the search?

Goal:
Reliable and performant termination criteria(s)

## 3 Definition of Termination Criteria

### Direct Termination Criteria

- **Maximal Generations / Time (T_1)**
  - a maximal number of generations / evaluations of objective function or a maximal time budget (absolute time, CPU time) is consumed

- **Running Mean (T_2)**
  - difference between the current best objective value \( f^* \) and the average of the best objective value of the last \( t_{\text{last}} \) generations is equal or less than a given threshold \( \varepsilon \)

- **Standard deviation (T_3)**
  - standard deviation of all objective values of the current generation is equal or less than \( \varepsilon \)

- **Best-Worst (T_4)**
  - difference between the best and the worst objective value of the current generation is equal or less than \( \varepsilon \)

### Hitting a Bound (T_5)

- **Phi (T_5)**
  - quotient of the best objective value \( f^* \) and the mean of all objective values of the current generation is equal or less than a given threshold \( \varepsilon \)

- **Kappa (T_6)**
  - quotient of the sum of all normalized distances \( d_i \) between all individuals of the current generation and \( \kappa_{\text{norm}} \) is equal or less than \( \varepsilon \)

- **T_7**
  - evaluates the spatial spreading of individuals of the current generation in the search space (normalized Euclidean distances \( d/d_\text{length of diagonal of search space} \))

### Cluster-Based Termination Criterion

- **ClusTerm (T_8)**
  - combines information of:
  - objective values and
distribution of individuals in the search space

### Procedure:

- Cluster analysis of the fittest individuals
- Determination of total amount \( N_i \) of individuals in clusters
- Terminate the search, when the change of the average of \( N_i \) is equal or less than \( \varepsilon \)

### Implementation:

- agglomerative cluster method
  - single linkage method
  - up to a maximal distance stage
- problem-specific distance measure
  (here Euclidean)

## 4 Evaluation and Guidelines

### Definition of evaluation criteria

- **Reliability**: guarantees termination within finite time
- **Performance**: no premature termination and no needless computation

### Evaluation of termination criteria

- **Reliability**:
  - T1 and T3 are reliable (by definition)
  - T8 is mostly reliable (terminated in all experiments, [2] for a conditional proof
- **Performance**:
  - T1: completely independent of the search
  - T2: efficiency depends on parameter \( t_{\text{last}} \) (i.e., \( t_{\text{last}} = 15 \))
  - T4: efficiency depends on choice of parameter \( \varepsilon \)

### Guidelines for the application of termination criteria

- Use at least one of the always reliable termination criteria T1 and T4
- Use the other (T3, T4 - T9) in conjunction with one of the reliable criteria T1 and T2
- Employ T1 and T4, to prevent needless computations in an inefficient state of the EA
- If lower bound or optimum is known employ T2 (combined with T1 for reliability)
- T4 - T9 can only be used with adaptive operators, the employment with discrete valued objective functions is not recommended

## 2 Termination

- Termination condition should avoid **needless computations** and prevent **premature termination**

  **If** TermCond **then** terminate EA **else** proceed EA

- **Needless computations** (efficiency of an EA is exhausted):
  - EA is degenerated to a random search
  - no significant improvement of the best objective value can be expected
- **Premature Termination**
  - EA terminates before the search process runs out

## 5 Summary

- Concise overview of a number of prominent termination criteria
- Definition of a new cluster-based termination criterion ClusTerm \( T_8 \)
  - combines information about objective values and distribution of individuals in the search space
  - first step to the development of intelligent termination criteria
- Systematic test of criteria using a set of discrete objective functions
- Guidelines for the practical employment of termination criteria
- Application to real-world problems proved successful
  - results of experiments verified by examples of Evolutionary Testing [5]
  - reliable automatic termination of multiple unattended optimization runs

- **Example implementation**:
  - “GEATbx: Genetic and Evolutionary Algorithm Toolbox for Matlab”
    http://www.geatbx.com/

### References