

On Termination Criteria of Evolutionary Algorithms

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1 Introduction

- Termination criteria decide on the end of an evolutionary search process (EA).
- Performance of an EA also depends on the determination of the appropriate point in time to terminate the search.

Problem:

What is the appropriate point in time to terminate the search?

Goal:

Reliable and performant termination criteria(s)

2 Termination

- Termination condition should avoid **needless computations** and prevent **premature termination**

If TermCond then terminate_EA else proceed_EA

- Needless computations (efficiency of an EA is exhausted):
 - EA is degenerated to a random search
 - no significant improvement of the best objective value can be expected
- Premature Termination
 - EA terminates before the search process runs out

3 Definition of Termination Criteria

Direct Termination Criteria

Maximal Generations / Time (T₁)

- a maximal number of generations / evaluations of objective function or a maximal time budget (absolute time, CPU time) is consumed

Hitting a Bound (T₂)

- best objective value f^* reaches or surpasses a bound f_{lim}

$$\text{TermCond}_2 = f_{lim} \geq f^*$$

Derived Termination Criteria

Running Mean (T₃)

- difference between the current best objective value f_t^* and the average of the best objective value of the last t_{last} generations is equal or less than a given threshold ϵ

Phi (T₆)

- quotient of the best objective value f_t^* and the mean of all objective values of the current generation is equal or less than a given threshold ϵ

Standard deviation (T₄)

- standard deviation of all objective values of the current generation is equal or less than ϵ

Kappa (T₇)

- quotient of the sum of all normalized distances d_{ij} between all individuals of the current generation and κ_{max} is equal or less than ϵ
- T₇ evaluates the spatial spreading of individuals of the current generation in the search space (normalized Euclidean distances d_{ij}/d , d :length of diagonal of search space)

Best-Worst (T₅)

- difference between the best and the worst objective value of the current generation is equal or less than ϵ

Cluster-Based Termination Criterion

ClusTerm (T₈)

combines information of:

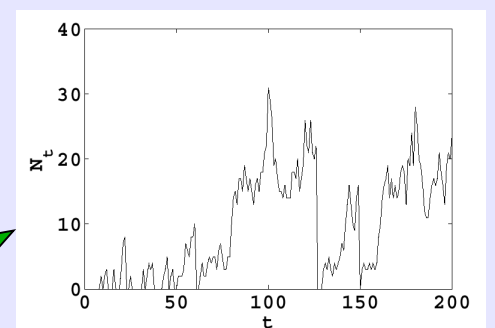
- objective values and
- distribution of individuals in the search space

Procedure:

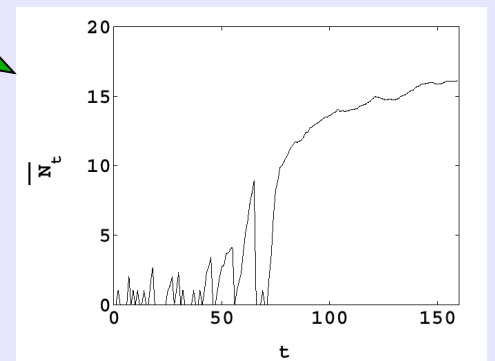
- Cluster analysis of the fittest individuals
- Determination of total amount N_t of individuals in clusters
- Terminate the search, when the change of the average of N_t is equal or less than ϵ

Implementation:

- agglomerative cluster method
 - single linkage method
 - up to a maximal distance stage
- problem-specific distance measure (here Euclidean)



Total amount N_t of fittest individuals in clusters during an optimization run



Average of N_t averaging over the last t_{last} generations (same run as above)

4 Evaluation and Guidelines

Definition of evaluation criteria

- Reliability:** guarantees termination within finite time
- Performance:** no premature termination and no needless computation

Evaluation of termination criteria

Reliability:

- T₁ and T₃ are reliable (by definition)
- T₈ is mostly reliable (terminated in all experiments, [2] for a conditional proof)
- T₂, T₄-T₇ are not reliable (terminate only under special circumstances)

Performance:

- assumes reliability, thus only T₁, T₃ and T₈ were examined
- T₁: completely independent of the search ⇒ in general T₁ inefficient
- T₃: efficiency depends on parameter t_{last} ⇒ robust setting of t_{last} possible (i.e., $t_{last} = 15$)
- T₈: efficiency depends on choice of parameter ϵ ⇒ robust setting of ϵ possible (i.e., $\epsilon = 0.1$)

Guidelines for the application of termination criteria

- Use at least one of the always reliable termination criteria T₁ and T₃
- Use the other (T₂, T₄ - T₈) in disjunction with one of the reliable criteria T₁ and T₃
- Employ T₈ and T₃ to prevent needless computations in an inefficient state of the EA
- If lower bound or optimum is known employ T₂ (combined with T₁ for reliability)
- T₄ - T₇ can only be used with adaptive operators, the employment with discrete valued objective functions is not recommended

5 Summary

- Concise overview of a number of prominent termination criteria
- Definition of a new cluster-based termination criteria ClusTerm T₈
 - combines information about objective values and distribution of individuals in the search space
 - first step to the development of intelligent termination criteria
- Systematic test of criteria using a set of discrete objective functions
- Guidelines for the practical employment of termination criteria
- Application to real-world problems proved successful
 - results of experiments verified by examples of Evolutionary Testing [5]
 - reliable automatic termination of multiple unattended optimization runs
- Example implementation:
 - “GEATbx: Genetic and Evolutionary Algorithm Toolbox for Matlab”
 - <http://www.geatbx.com/>

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